A Mathematical Model Analysis for Estimating Stock Market Price Changes

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Abstract

In this paper, different methods for estimation of parameters of Weibull distribution were examined using Mean Square Error (MSE) as a criterion for selecting the best model. The Method of Moments exceeded other methods. In the same circumstance, the estimated results were logically extended to form a matrix that would help in predicting different commodity price processes by exploring the properties of fundamental matrix solution where we obtained predicted stock prices and asset returns for 12 months. Finally, from the fundamental matrix system a theorem was developed and proved to show different levels of changes as it affects stock market in terms of short-run and long-run respectively.

Keywords: *Weibull distribution, stock market prices, Fundamental matrix solution and Estimation*

1. Introduction

In the real world of financial markets, investors and financial analysts are generally more interested in the profit or loss of the stock over a period of time, that is the changes in the price, than in the price self. Therefore, modeling a behavior of a stock exchange market can be made through its relative change of the unstable market variables in time so as to predict stock price fluctuation, advice investors and corporative owners who are working out for convenient ways to do business by issuing of stocks in their cooperation. Stock market fluctuations have a random walk property which needs to be examined. The rise and fall of stock market prices is caused by environmental factors such as bad governance, youth restiveness, civil war, epidemics (Covid-19) and lack of social amenities etc. The survival of investor may largely depends on previous knowledge about the behavior of stock market. That is why; it is imperative to consider stocks from Nigeria Stock Exchange (NSE) for the purpose of practical findings. However, the details of stock price modeling can be found in the following [6-16].

In two models method is the pretentious and effectual method in modeling stock prices in order to predict the future. The idea behind it is that the two models will be solved or analyzed

independently using stock market prices for prediction and detailed analysis for the purpose of model fitness. The two models, we shall explore in this paper are as follows; Weibull distribution and fundamental matrix solution. Weibull distribution was discovered by [1].It has two ranges of parameters. The application of Weibull distribution has been of mammoth interest to Scholars, Mathematicians and Statisticians alike. For instance, [2] workers on compartment analysis of methods of Estimating Weibull distribution. They applied three method of estimation such as maximum likelihood estimator, method of moments were selected as the best method bases on the selection criteria. In the same vein, [3] considered Weibull distribution for both analytics and numerical and results shows that the mean rank is the best method among the methods in the graphical and analytical procedures; on the numerical simulation studies the maximum likelihood estimation method (MLE) significantly outperformers other methods.
[4] after working on the method for estimating the parameters of weibull distribution, he presented both graphical and analytical method of estimating the Weibull distribution. Parameters discovered after computation of results that the method which gives the best estimates is the method of moments. Yunn-Kuangchu and Jau-chuanke examined the comparison of the two methods for Weibull parameters, one is the maximum likelihood method and the other is the least squares method. A numerical simulation study is carried out to understand the performance of the two methods. Based on sample root mean square errors they made a comparison between the two computation approaches and found out that the last square method significantly out performs the maximum likelihood when the sample size is small.

[2] Compared three methods of estimating 2-parameter Weibull distribution by using the Mean Squared Error(MSE) as the test criterion. Three methods used were Maximum Likelihood Estimator, Methods of moments and the Least Square Method. They concluded that the Method of moments was the best method based on the selection test criterion.

Clearly previous efforts have therefore considered similar problems but never investigated the combination of Weilbull distribution and analysis of fundamental matrix solution. In particular [2] and [3] considered the best estimation methods of estimating Weilbull parameters and the estimation of Weilbull parameter both analytical and numerical methods for predicting stock market price respectively. To the best our knowledge this is the first kind to combine Weibull distribution and fundamental matrix solution to predict stock returns.

This paper is arranged as follows: Section 2.1 presents Mathematical Formulation of Weibull distributions, methods of estimation is seen in Section 2.2, Section 2.3 comparison of estimation methods,Section2.4 a fundamental matrix solution of stock market prices, Section 2.5 presents analysis of fundamental matrix solution of stock returns while Section 3 is discussions of results and the paper is concluded in Section 4.

2.1 Mathematical Formulation of Weibull distributions

let S_1 S_2…, S_N be a random sample of size N from a population, ∍ $S_i \in □$, $S_i > 0$.
The general form of a three parameter Weibull probability density function (pdf) is given by

$$F(x) = \frac{\beta}{\propto}\left(\frac{x_1 - V}{\propto}\right)^{\beta - 1} \exp\left\{\left(\frac{x_1 - u}{\alpha}\right)^{\beta}\right\} xv \geq 0; \alpha, \beta > 0 \qquad (1)$$

Where; x_1 is the data vector at time; β is the shape parameter; ∝ is the scale parameter that indicates the spread of the distribution of sampled data and v is the location parameter.

The cumulative distribution function (cdf) of the weibull distribution is mathematically given as:

$$F(x_1) = 1-\exp\left\{\left(\frac{x_1 - v}{\alpha}\right)\right\} \quad (2)$$

Incase of v=0, the probability distribution function in equation (1) reduces to equation (2)

$$F(x_1) = \left\{\left(\frac{\beta}{\alpha}\right)\left(\frac{x_1}{\alpha}\right)\right\}^{\beta-1} \exp\left\{-\left(\frac{x_1}{\alpha}\right)^{\beta}\right\} \; x \geq 0,\ \alpha,\ \beta > 0 > 0 \quad (3)$$

otherwise with a corresponding cumulative distribution function as

$$F(x_1) = \begin{cases} 1-\exp\left\{-\left(\frac{x_2}{\alpha}\right)^{\beta}\right\}, x \geq 0 \\ 0, \text{otherwise} \end{cases} \quad (4)$$

The mean and variance of the weibull are E(x) = αr (1 + $^1/_\beta$) and v(x) = α^2 [r(1 + $^2/_\beta$) = ⌈^{2} (1 + $^1/_\beta$)] respectively, where r(n)gamma function evaluated at n. The various analytical methods in estimating weibull parameters are:

2.2Methods of Estimation

The methods of estimation are as follows(a)Maximum likelihood Estimator (MLE)(b) Methods of Moments (MOM) and (c)The Least Square Method (LSM)

2.2.1 Maximum Likelihood Estimator (MLE)

Let $x_1, x_2 \ldots x_n$ represent a random sample of size n drawn from a population with probability density function f (x,λ) where $\lambda = (\beta, \alpha\,)$is an unknown vector of parameters. The likelihood function is defined as: $L = f(\alpha, \beta) = \prod_{i=1}^{n} f(x_i, \lambda)$ (5)

The maximum likelihood of $\lambda = (\beta, \alpha)$,maximum L or equivalently, the logarithm of L when

$$\frac{\delta InL}{\delta \lambda} = 0 \quad (6)$$

Where solutions that are not function of the sample values.$x_1, x_2 \ldots\ x_n$are not admissible. Now are apply the maximum likelihood estimator to estimate the parameters of weibull distribution, namely ∝ and β respectively where y = 0.

Consider the Weibull probability density function given in equation (5) the likelihood function will be given as:

$$L\ (x_1, x_2 \ldots\ x_n : \beta, \propto) = \prod_{i=1}^{n} = 1\left(\frac{\beta}{\propto}\right)\left(\frac{x_i}{\propto}\right)^{\beta-1} - e\left\{\left(\frac{x_i}{\propto}\right)^{\beta}\right\}$$

$$= \left(\frac{\beta}{\propto}\right)^{n}\left(\frac{1}{\propto}\right)^{\beta-1}\sum_{n-1}^{n} \propto i(\beta-1)e - (x_i)^{\beta-1}\ e - \left(\frac{x_i}{\propto}\right)^{\beta}$$

$$= \left(\frac{\beta}{\propto}\right)^{n}\left(\frac{1}{\propto}\right)^{n\beta-n}\sum_{i1}^{n}\alpha i\,(^{\beta-1})_{e} - \sum_{n=1}^{n}\left(\frac{xi}{\alpha}\right)^{\beta} \qquad (7)$$

Taking the algorithms of both sides and differentiating partially with respect to β and ∝ in turn and equating to zero, we obtain the estimating equations as follows:

$$\frac{\delta I_{nL}}{\delta\beta} = \frac{n}{\beta} \div \sum{}^{n}_{i=1}\sum{}^{\beta}_{i}\ I_n x_i = 0 \qquad (8)$$

$$\frac{\delta I_{nL}}{\delta\propto} = \frac{n}{\propto} + \frac{1}{\alpha^2}\sum_{L=1}^{n} x_i^{\beta} = 0 \qquad (9)$$

$$\frac{\delta I_{nL}}{\delta\propto} = \frac{-n}{\propto} + \frac{1}{\propto^2}\sum_{n=1}^{n} x^{\beta} = 0 \qquad (10)$$

so substituting (10) in (11) gives

$$I_n x_i \frac{-1}{\beta} + \frac{1}{n}\sum_{i=1}^{n} Inx_i = 0 \qquad (11)$$

$$\text{Hence } \beta MLE = \sum_{L=1}^{n} x_i \overset{n}{P} \sum_{i=1}^{n} In\frac{(xi)}{n} \qquad (12)$$

∝ is now estimated using equation (10)

$$-\propto^2{}_n \div \propto \sum_{i=1}^{n} x_i\,\beta = 0 \qquad (13)$$

So that:

$$\overset{n}{\alpha}_{MLE} = \sum_{L=1}^{n} x_i \overset{n}{P} \sum_{i=1}^{n} x_i^{\hat{\beta}} \tag{14}$$

2.2.2 Method Of Moments (MOM)

Let $x_1, x_2 \ldots x_n$ be a random sample and then an unbased estimator for the K^{th} moment is given by:

$$m_k = \frac{n}{\alpha} \sum_{i=1}^{n} x_i^k \text{ i} \tag{15}$$

Where $\hat{m}_k$ denotes the estimate of K^{th} moment. In weibull the K^{th} moments follows from equation (16).

$$\mu_k = \left(\frac{1}{\propto\beta}\right)^{-x/p} - {}^{k}/_{\beta}\, \Gamma(1 + \frac{k}{\beta}) \tag{17}$$

Where Γ is a gamma function evaluated at the value of $(1 + \frac{1}{\beta})$ which

Provides the values Γ (k) at any value of k from equation (15), we can find the 1st and 2nd moments as follows.

$$\hat{M}_1 = \hat{\mu}_1 = (\frac{1}{\propto})^{1/\beta}\Gamma(1+\frac{1}{\beta}) \tag{17}$$

$$\hat{M}_L = \mu_1 + \sigma^2 = [\frac{1}{\propto}]^{2/\beta} \{[\Gamma(1+\frac{2}{\propto}) - \Gamma(1+\frac{1}{\beta})]^2\} \tag{18}$$

Dividing m_1 by the square of m_2, we get an expression which is a function of β only .

$$\frac{\mu_i}{\sigma^2+\mu^2} = \frac{\Gamma\left(1+\frac{1}{\beta}\right)\Gamma\left(1+\frac{1}{\beta}\right)}{\Gamma\left(1+\frac{2}{\beta}\right)} \tag{19}$$

Where $\hat{\mu} = \sum_{i=1}^{n} In(\frac{Si}{s_{i-1}}) = E(x_i) = \frac{1}{n}\sum_{t=1}^{n} x_i$, $\sigma^2 = E(x^2{}_i) - (E(x_i))^2$

And $Z = {}^{1}/_{\beta}$

Equation (18) is transformed in order to estimate β and ∝ repectively

$$\frac{\mu^2}{\sigma^2+\mu^2} = \frac{\Gamma\left(1+\frac{z}{\beta}\right)\Gamma\left(1+\frac{z}{\beta}\right)}{\Gamma(1+\frac{2z}{\beta})} \tag{20}$$

The value of the scale parameter ∝mom can be estimated thus:

$$\hat{\propto}_{Mom} = \frac{\hat{\mu}}{\Gamma\left(1+\frac{1}{\hat{\beta}}\right)} \quad (21)$$

Where $\hat{\mu}$ is the mean of the original data

2.2.3 Least Squares Method (LSM)

Here we assume that there is a linear relationship between two values considering.

$$Y = \propto + \beta x_1 \quad (22)$$

$$Y = In\,[In\,(\frac{1}{1-F(T)})];\ m = \beta,\ x = Inx \text{ and } b = \beta In\propto \quad (23)$$

Assume that a set of data pairs (y_1, y_2), (x_1,x_2) … (x_n,x_y) were obtained and plotted.

Following the least squares concept which minimizes the vertical distance between the data points and the straight line fitted to the data, the best fitting line to this data is the straight line

$$Y = \propto + \beta x$$

Such that $\sum^{n}_{t=1}(y_i - \propto + \beta_{xi})^2 = m_{In}(\propto,\beta) = \sum^{n}_{t=1}(y_i - \propto + \beta x_i)^2$ (24)

Where $\hat{\alpha}$ and $\hat{\beta}$ we let $Q = \sum^{n}_{t=1}(y_t - \propto + \beta_{xt})^2$ and differentiating Q with respect to β and equating to zero yields the following systems of equations:

$$\frac{\delta Q}{\delta \beta} = 2\sum^{n}_{t=1}(y_i - \propto + \beta x_i)^2 = 0 \quad (25)$$

$$\frac{\delta Q}{\delta \beta} = 2\sum^{n}_{t=1}(y_i - \propto + \beta x_i)^2 x_i = 0 \quad (26)$$

Expanding and solving equation (24) and (25) simultaneously, we have:

$$\hat{\beta}_{Lsm} \frac{\sum^{n}_{t=1} x_i y_i - \sum^{n}_{t=1} x_i \sum^{n}_{t=1} y_i}{\sum^{n}_{t=1} x_i - \frac{(\sum^{n}_{t=1} xt)}{n}}) \quad (27)$$

$$\text{And } \hat{\propto}_{LSM} = \frac{\sum^{n}_{t=1} y_i}{n} - \hat{\beta}\frac{\sum^{n}_{t=1} x_i}{n} = y_i - \beta x i \quad (28)$$

2.3 Comparison of Estimation Methods

We have derived the three analytical methods for estimating Weibull distribution such as Maximum Likelihood Estimator, Method of Moment and Least Square Method.

The Mean Square Errors shall be used as a criterion for selection.

The mean squared error (MSE) criterion is given by:

$$\text{MSE} = \frac{1}{n}\sum_{i=1}^{n}[\hat{f}(x_i) - F((x_i)]^2 \tag{29}$$

Where f $((\hat{x}_i)$ is obtained by substituting the estimates of ∝ and β in (4) (for each method) while $F(x_i) = {}^{i}/_{n}$ is the empirical distribution function. The method with the mimmum mean squared error (MMMSE) becomes the best methods for the estimation of weibull parameters among the candidate methods.

2.4 A Fundamental Matrix of Stock Market prices

Suppose $x_1, x_2, \cdots, x_n$ is a fundamental set of solution of the homogeneous system(stock returns) on an interval I then its general solution on the interval is given as:

$$X = c_1x_1 + c_2x_2 + \cdots + c_nx_n$$

$$= c_2\begin{pmatrix} x_{11} \\ x_{21} \\ \vdots \\ x_{n1} \end{pmatrix}_1 + c_2\begin{pmatrix} x_{12} \\ x_{22} \\ \vdots \\ x_{n2} \end{pmatrix} + \cdots + c_n\begin{pmatrix} x_{1n} \\ x_{2n} \\ \vdots \\ x_{nn} \end{pmatrix}, = \begin{pmatrix} c_1x_{11} + & c_2x_{12} + & \cdots + & c_nx_{1n} \\ c_1x_{21} + & c_2x_{22} + & \cdots + & c_nx_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ c_1x_{n1} + & c_2x_{n2} + & \cdots + & c_nx_{nn} \end{pmatrix}$$

Definition 1: Let

$$X_1 = \begin{pmatrix} x_{11} \\ x_{21} \\ \vdots \\ x_{n1} \end{pmatrix}, X_2 = \begin{pmatrix} x_{12} \\ x_{22} \\ \vdots \\ x_{n2} \end{pmatrix}, \cdots, X_n = \begin{pmatrix} x_{1n} \\ x_{2n} \\ \vdots \\ x_{nn} \end{pmatrix}$$

Be a fundamental set of n stock returns of solution vectors of the homogeneous system on an interval I. The stock return matrix is as follows:

$$A(t) = \begin{pmatrix} x_{11} & x_{12} & \cdots & x_{11} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ x_{n1} & x_{n2} & \cdots & x_{nn} \end{pmatrix} \tag{30}$$

Is a fundamental matrix of stock returns.

To solve (30), assume a solution of the form

$$x = Ae^{rt} \text{ and } y = Be^{rt} \quad (31),$$

where A, B, r, are constants to be determined . Solving the system above gives three possibilities of the roots

(i) Two distinct roots r_1, r_2 ie $r_1 \neq r_2$,
(ii) Two equal roots $r_1 = r_2$
(iii) Complex conjugate roots $r_1 = a \pm ib$

The system produces the Eigen-values and their corresponding Eigen-vectors which serves as the independent solutions

2.5 Analysis of fundamental matrix solution of stock returns

We took the stock price estimates of MLE, and LSM in Table 1 to form a complete 2×2 matrix as seen below:

$$\sum = \begin{pmatrix} 0.6992 & 0.7399 \\ 0.4371 & -0.3268 \end{pmatrix} \tag{32}$$

Solving the above matrix gives eigenvalues and eigenvectors respectively

$$\lambda_1 = 0.9521,\ \lambda_2 = -0.5797,\ K_1 = \begin{pmatrix} 1.0001 \\ 0.3418 \end{pmatrix},\ K_2 = \begin{pmatrix} -3.5847 \\ -1.7285 \end{pmatrix}$$

The eigenvectors are summarized as follows:

$$\begin{pmatrix} -1.0045 \\ -0.1644 \end{pmatrix}$$

The vectors are:

$$c_1 = \begin{pmatrix} 1.0001 \\ 0.3418 \end{pmatrix} e^{0.9521t} = \begin{pmatrix} 1.0001e^{0.9521t} \\ 0.3418e^{0.9521t} \end{pmatrix} \ and \ c_2 = \begin{pmatrix} -3.5847 \\ -1.7285 \end{pmatrix} e^{-0.5797t} = \begin{pmatrix} -3.5847e^{-0.5797t} \\ -1.7285e^{-0.5797t} \end{pmatrix}$$

This is the fundamental set of solution of the system on the $-\infty < t < \infty$ .The fundamental matrix of system becomes.

$$X(t) = c_1 k_1 + c_2 k_2$$

$$c_1 \begin{pmatrix} 1.0001 \\ 0.3418 \end{pmatrix} e^{0.9521t} + c_2 \begin{pmatrix} -3.5847 \\ -1.7285 \end{pmatrix} e^{-0.5797t}$$

$$\begin{pmatrix} X_1(t) \\ X_2(t) \end{pmatrix} = \begin{pmatrix} c_1 1.0001e^{-0.9521t} - c_2 3.5847e^{-0.5797t} \\ c_1 0.3418e^{-0.9521t} - c_2 1.7285e^{-0.5797t} \end{pmatrix}$$

This in turn yields more familiar statement

$$\left.\begin{aligned} X_1(\mathrm{t}) &= \mathrm{c}_1\, 1.0001 e^{0.9521t} - \mathrm{c}_2\, 3.5847 e^{-0.5797t} \\ X_2(\mathrm{t}) &= \mathrm{c}_2\, 0.3418 e^{0.9521t} - \mathrm{c}_2\, 1.7285 e^{-0.5797t} \end{aligned}\right\} \tag{33}$$

$$X(t) = \begin{pmatrix} 1.0001 e^{0.9521t} & -3.5847 e^{-0.5797t} \\ 0.3418 e^{0.9521t} & -1.7285 e^{-0.5797t} \end{pmatrix} \tag{34}$$

Considering the levels of changes as a of stock market price fluctuations. We take the rate of change of (34) with respect to time, t gives

$$X'(t) = \begin{pmatrix} 0.9522 e^{-0.9521t} & 2.0781 e^{-0.5797t} \\ 0.3254 e^{-0.9521t} & 1.0020 e^{-0.5797t} \end{pmatrix} \tag{35}$$

Also we consider the fundamental matrix solution of(35) which contains four stock prices of four different trading weekly(7) days; in a month gives 28 trading days. Now let $X_i, i = 1, 2, ..., \mathrm{T}$ be the estimated stock prices for T trading days. Thus, when t=1,2,…,10 in (35) gives the following predicted stock prices.

$$\sum_{t=1}^{4} X'(1) = \begin{pmatrix} 2.4673 & 1.1638 \\ 0.8432 & 0.5612 \end{pmatrix} = 5.0356, \; \sum_{t=1}^{4} X'(2) = \begin{pmatrix} 6.3931 & 0.6518 \\ 1.6863 & 0.3143 \end{pmatrix} = 9.0455$$

$$. \sum_{t=1}^{4} X'(3) = \begin{pmatrix} 7.4019 & 0.3651 \\ 2.5294 & 0.1760 \end{pmatrix} . = 10.4724 + \cdots + \sum_{t=1}^{4} X'(12) = \begin{pmatrix} 29.6075^{-} & 13.9605 \\ 10.1179 & 6.7342 \end{pmatrix} = 60.4261$$

Therefore, for simplicity and without loss of generality we define each month of return and percentage effects as:

$$R_t = \frac{\sum_{i=1}^{4} X_i}{\sqrt{\tau}}, i = 1, 2, 3, 4 \tag{36}$$

Where $\tau \, is \sqrt{28}$ trading days, while the percentage effects of each stock is defined as follows:

$$\beta_k = R_t \times 100 \tag{37}$$

2.5. 1The Independent Solution of Fundamental Matrix of Capital Investments

Realistically, stock trading is a process whose domain is defined on a probability space. Suppose the independent solution of fundamental matrix of capital investments follows continuous time stochastic process, then physical reality requires that the model of independent solution undergoes levels of changes until something happens in the process such that the probability considerations are satisfied.

Theorem 1: Let a continuous random variable t have the independent solution of fundamental matrix of capital investment (33) whose dynamics has levels of changes and follows exponential distribution function:

$$f\left(X_1(\mathrm{t}), X_2(\mathrm{t})\right) = \begin{cases} c_1 1.0001e^{0.9521t} - c_2 3.5847e^{-0.5797t} & for\ t > 0 \\ c_1 0.3418e^{0.9521t} - c_2 1.7285e^{-0.5797t} & for\ t > 0 \\ 0 & for\ t \le 0 \end{cases}$$

Proof:

We want to show that our proposed solution of (33) is continuous time stochastic process which undergoes some levels of probabilistic concepts.

We determine the mean of the function as

$$E(t) = -c_1 0.9946\int_0^\infty te^{-0.3714t}dt - c_2 1.0045\int_0^\infty te^{-0.0863t}dt$$

Using Nedu's method of integration by parts gives

$$= \left[\left(c_1 0.9946t\frac{e^{-0.3714t}}{0.3714} + c_1 0.9946t\frac{e^{-0.3714t}}{0.3714}\right) + \left(c_2 1.0045t\frac{e^{-0.0863t}}{0.0863} + c_2 1.0045t\frac{e^{-0.0863t}}{0.0863}\right)\right]$$

$$= c_1 0.9946\left(\frac{te^{-0.3714t}}{0.3714} + \frac{e^{-0.3714t}}{0.3714}\right) + c_2 1.0045\left(\frac{te^{-0.0863t}}{0.0863} + \frac{e^{-0.0863t}}{0.0863}\right)\Big|_0^\infty$$

$$= c_1 0.9946\left(\frac{1}{0.3714}\right) + c_2 1.0045\left(\frac{1}{0.0863}\right)$$

$$= 2.6780c_1 + 11.6340c_2$$

The second moment of exponential distribution function is defined thus:

$$E(\mathrm{t}^2) = -c_1 0.9946\int_0^\infty t^2 e^{-0.3714t}dt - c_2 1.0045\int_0^\infty t^2 e^{-0.0863t}dt$$
$$= -c_1 0.9946\left(-\frac{t^2e^{-0.3714t}}{0.3714} - \frac{2te^{-0.3714t}}{0.3714} - \frac{2e^{-0.3714t}}{0.3714}\right) - c_2 1.0045\left(-\frac{t^2e^{-0.0863t}}{0.0863} - \frac{2te^{-0.0863t}}{0.0863} - \frac{2e^{-0.0863t}}{0.0863}\right)$$
$$= c_1\left[\frac{0.9946t^2e^{-0.3714t}}{0.3714} + \frac{1.9892te^{-0.3714t}}{0.3714} + \frac{1.9892e^{-0.3714t}}{0.3714}\right] + c_2\left[\frac{1.0045t^2e^{-0.0863t}}{0.0863} + \frac{2.009te^{-0.0863t}}{0.0863} + \frac{2.009e^{-0.0863t}}{0.0863}\right]\Big|_0^\infty$$
$$= 5.3560c_1 + 23.2793c_2$$

From first and second moments we have the variance as:

$$Var(\mathrm{t}) = E(\mathrm{t}^2) - E(\mathrm{t})^2$$
$$= 2.6780c_1 + 11.6340c_2 - 28.6867c_1 - 565.4551c_2$$

collecting like terms yields

$$-26.0087c_1 - 553.8211c_2$$

Clearly the conditions are satisfied, therefore the claim is true. This is to show that our proposed solution obey some physical considerations

3. Results and discussions

In this Section we present the computational results for the problems formulated in Section 2

Table 1:Comparison of stocks Estimates of Weilbull distribution

Method of Estimation	$\hat{\alpha}$	$\hat{\beta}$	MSE
MLE	0.6992	0.7399	2.9799 x 10^{-5}
MOM	0.4420	2.5284	2.1849 x 10^{-6}
LSM	0.4371	-0.3268	2.9799 x 10^{-5}

The estimates of the parameters based on theoretical procedures described in Section 2.2 are presented on Table1.The shape parameter lies within the interval (0,3) which implies, as indicated that the function decreases exponentially. We ranked the performance of the methods based on the least MSE criterion. The Method of Moments (MOM) showed the least values when compared to MLE and LSM respective see Table 1

Table 2: Fundamental matrix solution of stock quantities

Months (t)	α_{11} -Stock Prices	α_{12} - Stock Prices	α_{21} - Stock Prices	α_{22} - Stock Prices	Predi Stock Prices	Return rate (R_t)	$\beta_k = R_t \times 100$
1	2.4673	1.1639	0.8432	0.5612	5.0356	0.9589	95.89
2	6.3931	0.6518	1.6863	0.3143	9.0455	1.7225	172.25
3	7.4019	0.3651	2.5294	0.1760	10.4724	1.9942	199.42
4	9.8692	0.2045	3.3726	0.09859	13.5449	2.5792	257.92

5	12.3364	0.1145	4.2158	0.0552	16.7219	3.1842	318.42
6	14.8037	0.0640	5.0590	0.03092	19.9576	3.8004	380.04
7	17.2710	8.1471	5.9021	3.9283	35.2485	6.7121	671.21
8	19.7383	9.3109	6.7454	4.4895	40.2841	7.6709	767.09
9	22.2056	10.4749	7.5884	5.05068	45.319	8.6298	862.98
10	24.6729	11.6387	8.4316	5.6119	50.3551	9.5887	958.87
11	27.1402	12.8026	9.2747	6.1731	55.3906	10.5476	1054.76
12	29.6075	13.9665	10.1179	6.7342	60.4261	11.5064	1150.64

In Table 2, it can be seen that a little increase in time of trading increases the predicted stock prices, return rates and percentage effects of stock returns. This indicates that as years or months keeps moving forward, the value of assets continue to increase. This remarks are beneficial to investors who speculates(as in European option) in investing huge amount of money in lands, houses and shares etc. Also the changes in the stock quantities formation stipulate that stock price is a stochastic process, hence is time dependent.

However,two eigenvalues represents the total amount of predicted stocks . The $\lambda_1 = 0.9521$ is greater than zero which is an indication of investment return in the side of the investment whose aim and passion is to maximize profit. So $\lambda_2 = -0.5797$ represents the levels of losses made all through the trading treading days by an investor. The eigenvectors determines the direction of the stock market prices in terms of changes

4 Conclusions

This paper investigated different methods for estimation of parameters of Weibull distribution, using Mean Square Error (MSE) as a criterion for selecting the best model. The result showed that Method of Moments exceeded other methods. In the same vain, the estimated results were logically extended to form a matrix that would help in predicting different commodity price processes by exploring the properties of fundamental matrix solution where we obtained predicted stock prices and asset returns for 12 months which reveals that: increase in time of trading increases the predicted stock prices, return rates and percentage effects of stock returns.

Finally, from the fundamental matrix system a theorem was developed and proved to show different levels of changes as it affects stock market in terms of short-run and long-run respectively.We shall be considering the uniqueness of the stock quantities in the next study.

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